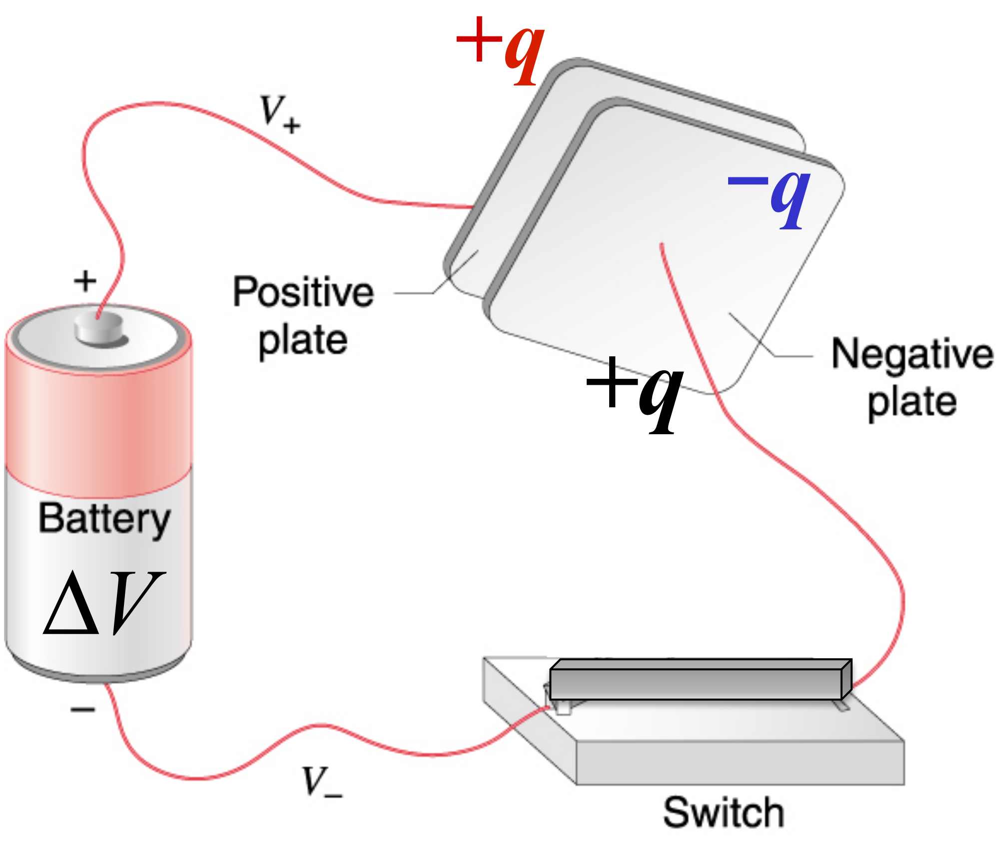
Chapter 23: Electrostatic Energy & Capacitance
Thursday September 22nd

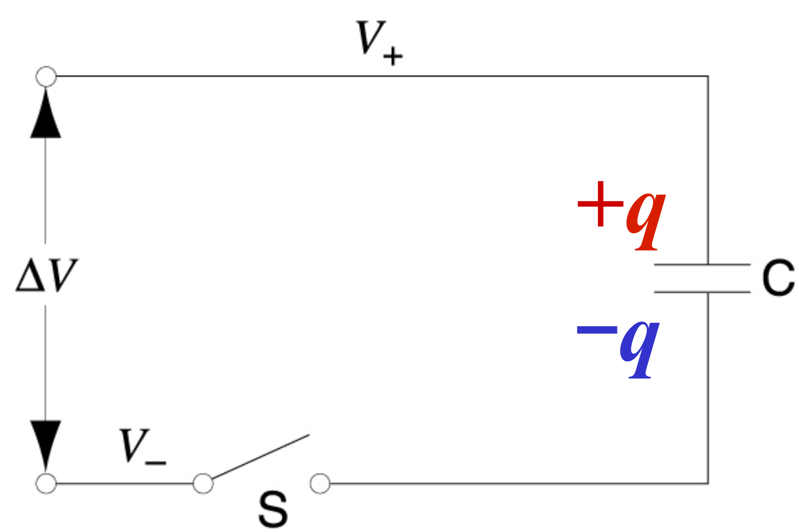
- Capacitance
  - Definition
  - Review parallel plate example
- Electrostatic potential energy stored in capacitors
  - Analogy with springs
  - Constant charge/constant voltage
- Capacitors in series and parallel
  - Demonstration and example
- Dielectrics and capacitance
  - Demonstration

Reading: up to page 393 in the text book (Ch. 23)

Capacitors

- Used to store energy in electromagnetic fields [in contrast to batteries (chemical cells) that store chemical energy].
- Capacitors can release electromagnetic energy much, much faster than chemical cells. They are thus very useful for applications requiring very rapid responses.





Capacitors

- The transfer of charge from one terminal of the capacitor to the other creates the electric field.
- Where there is an electric field, there must be a potential difference, leading to the following definition of capacitance *C*:

$$C = \frac{Q}{\Delta V} \qquad \text{or} \qquad Q = C\Delta V$$

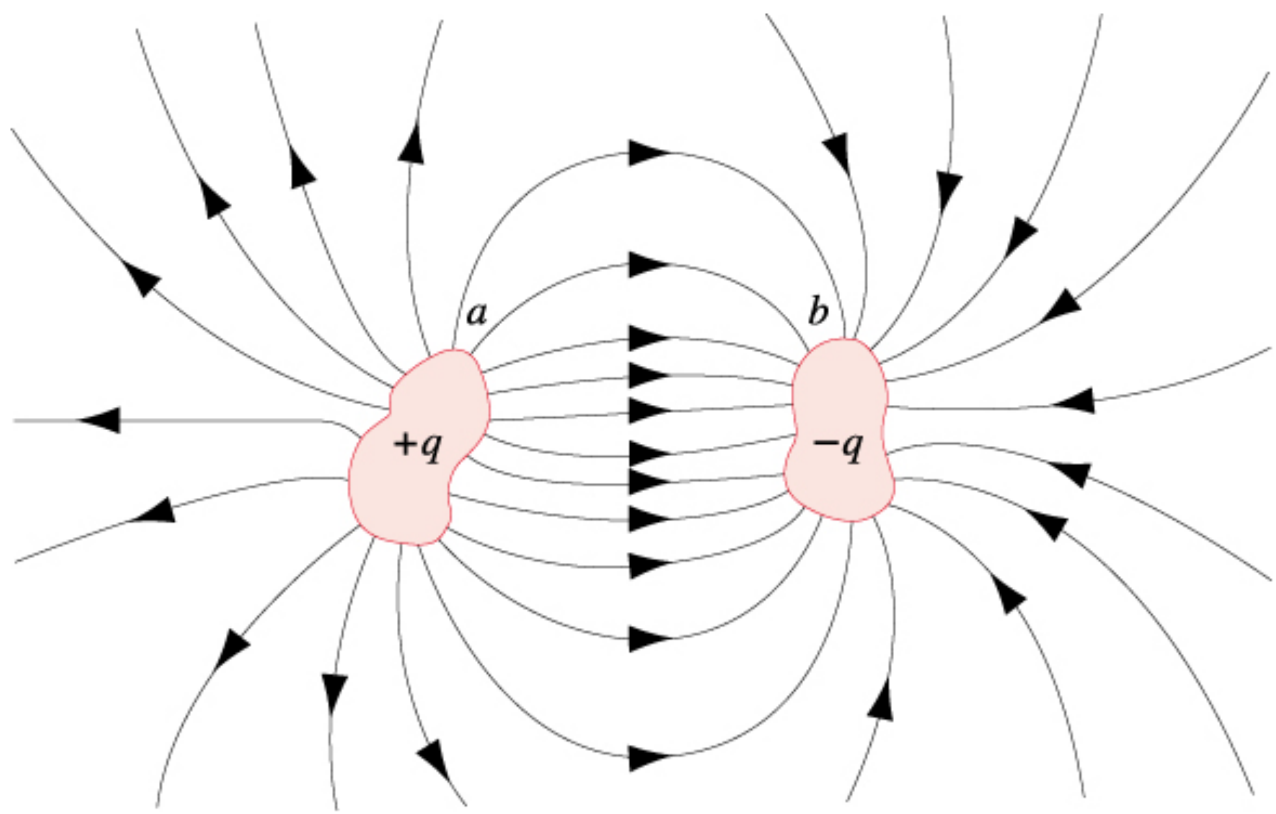
- *Q* represents the magnitude of the excess charge on either plate. Another way of thinking of it is the charge that was transferred between the plates.

SI unit of capacitance: 1 farad (F) = 1 coulomb/volt

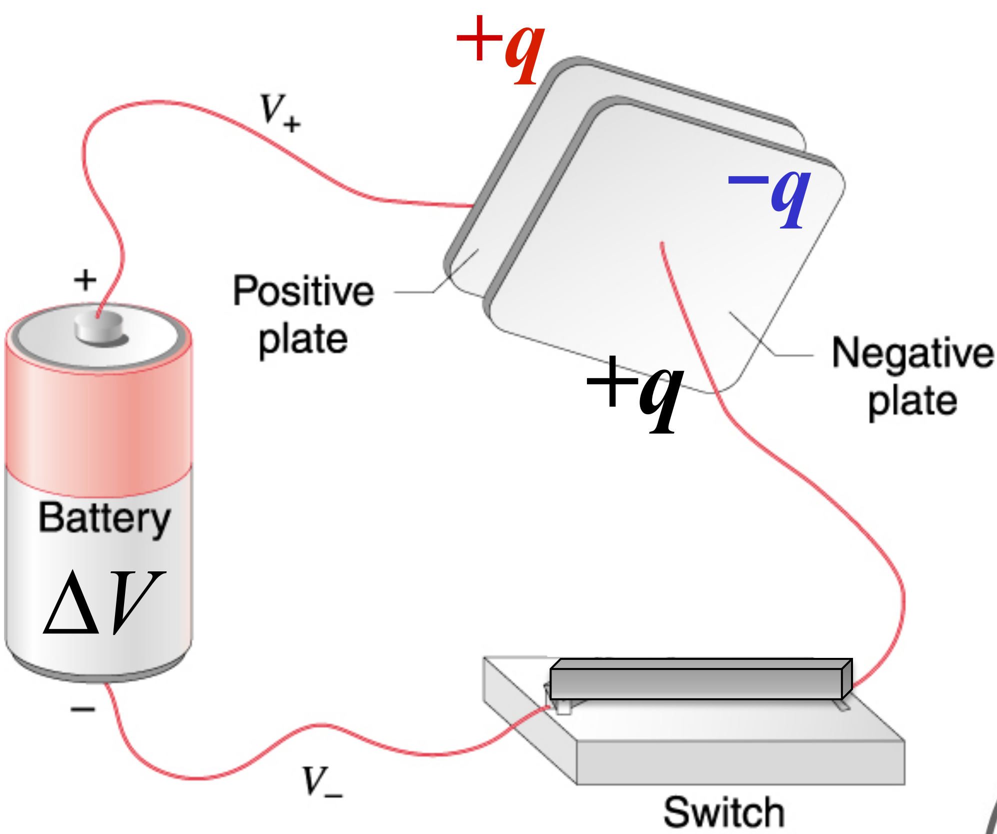
(after Michael Faraday)

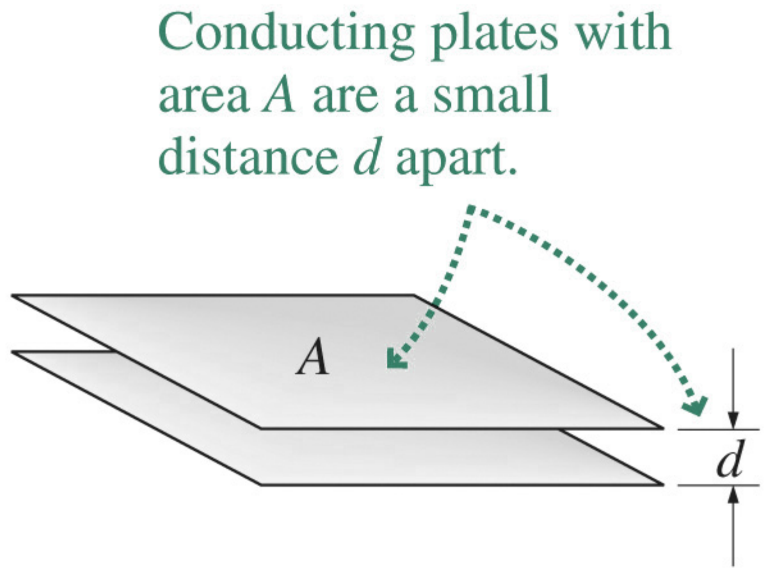
Capacitors

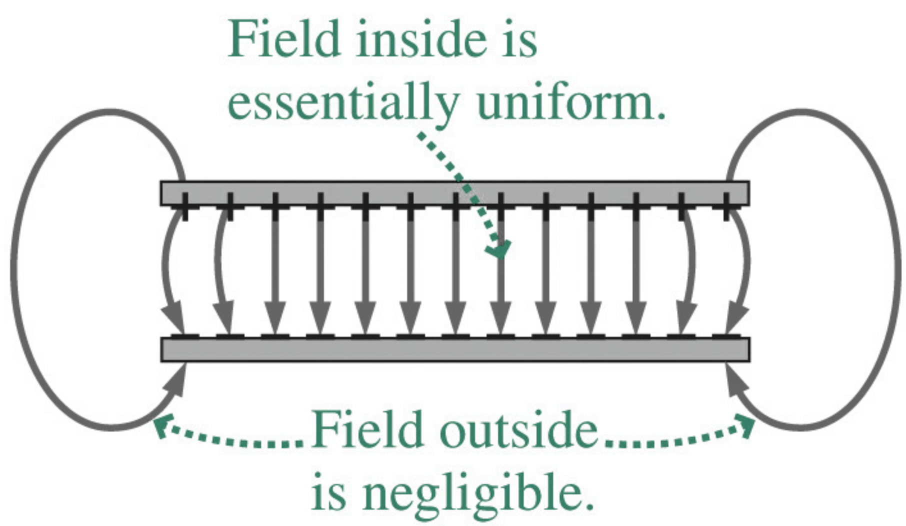
- The energy really is stored in the electromagnetic fields.
- In fact, these fields possess energy and momentum, so you might think of the capacitor as a fly-wheel, though it is more common to think of capacitors as the electrical analog of springs (as will become apparent in a moment).



The Parallel Plate Capacitor (standard example)

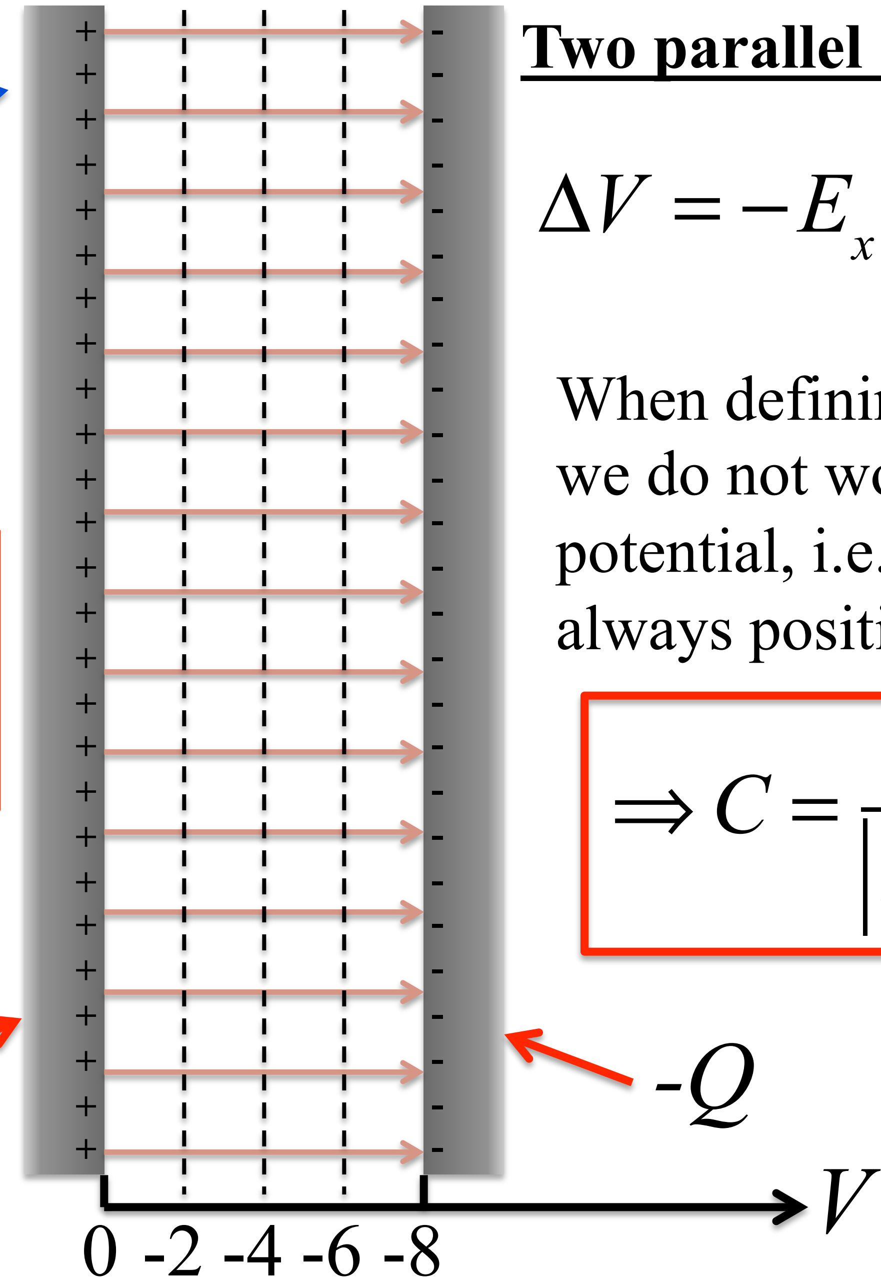






$$C = \frac{Q}{\Delta V}$$

The Parallel Plate Capacitor (standard example)



$$\pm\sigma = \frac{\pm Q}{A}$$

$$E_x = \frac{\sigma}{\varepsilon_o} = \frac{Q}{A\varepsilon_o}$$

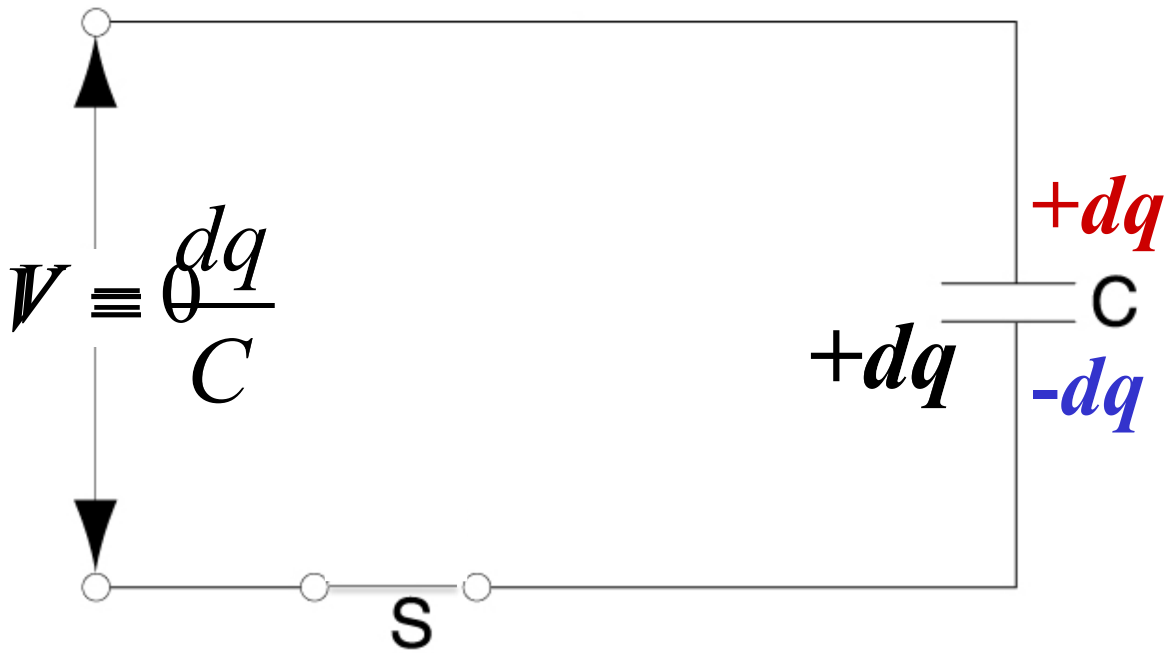
Two parallel conducting plates

$$\Delta V = -E_x \Delta x = -\frac{Q}{A\varepsilon_o} d$$

When defining capacitance, we do not worry about sign of potential, i.e., capacitance is always positive

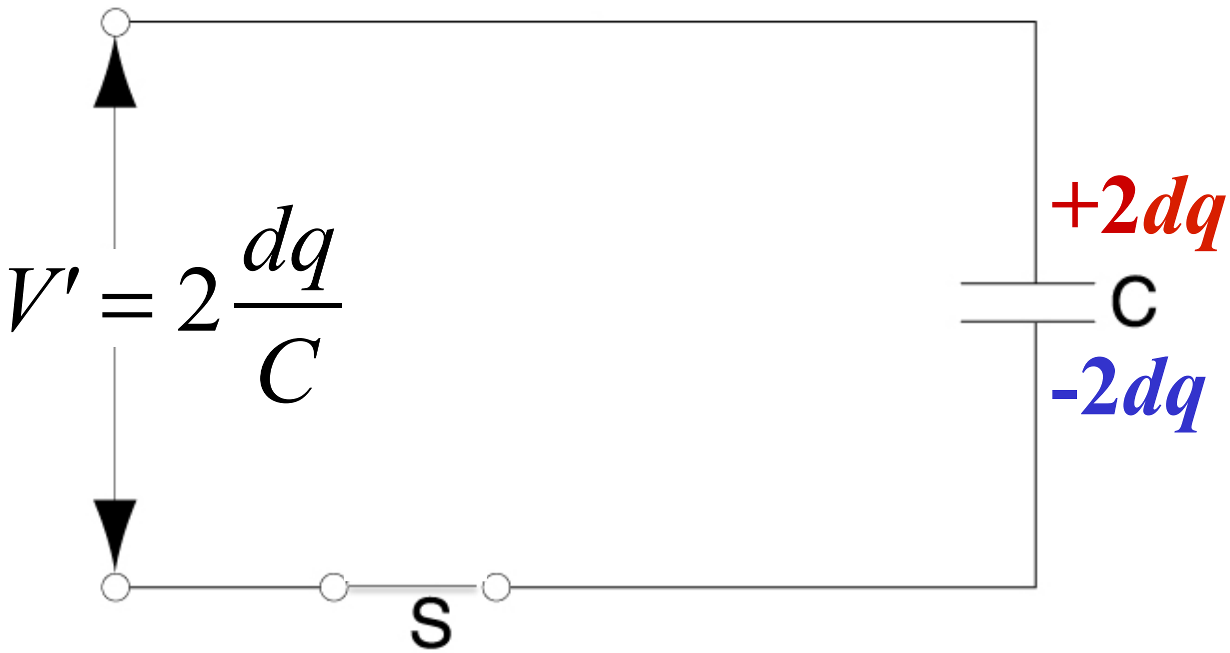
$$\Rightarrow C = \frac{Q}{|\Delta V|} = \frac{\varepsilon_o A}{d}$$

Energy stored in a Capacitor



$$dU = dq \times V$$

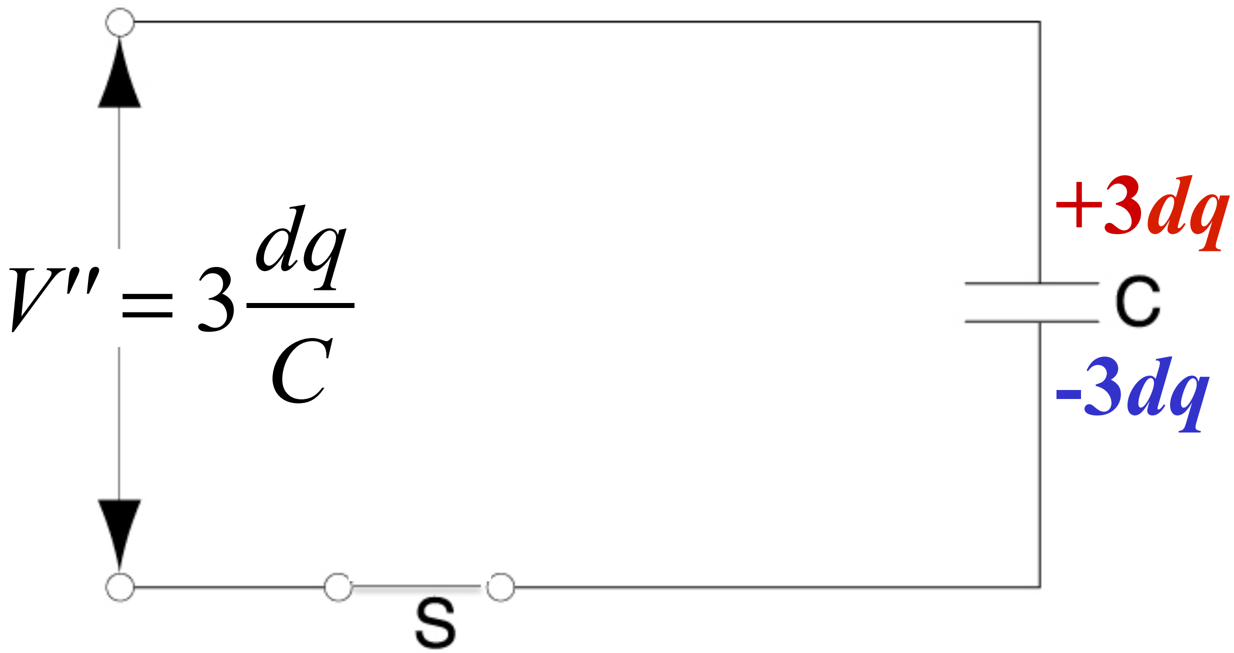
Energy stored in a Capacitor



$$U = dq \times V$$

$$+ dq \times V'$$

Energy stored in a Capacitor

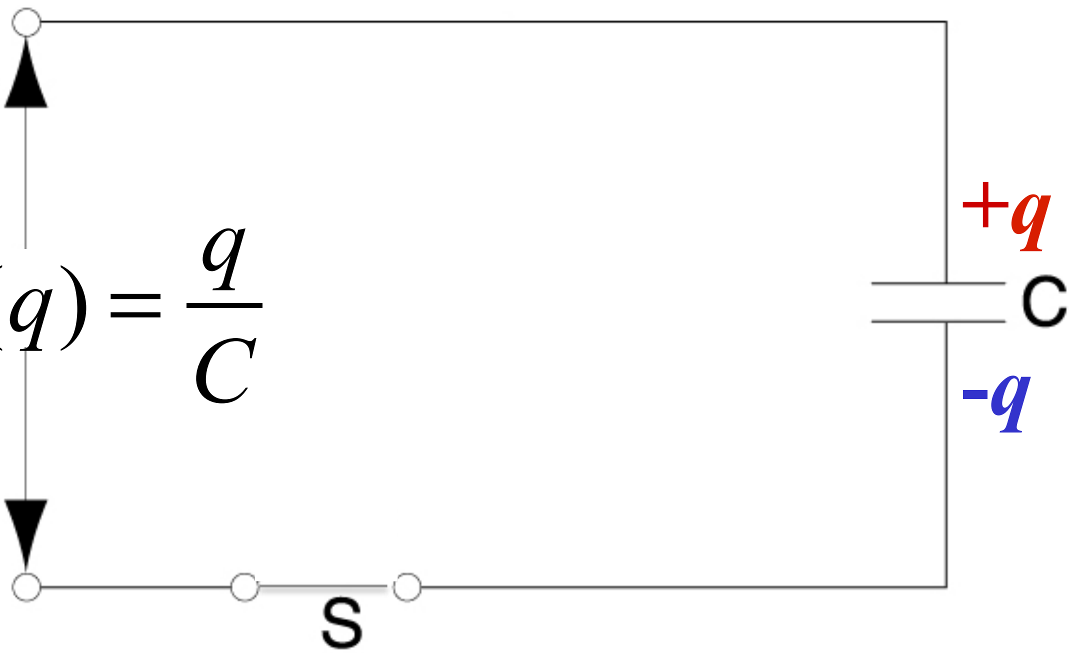


$$U = dq \times V$$

$$+ dq \times V'$$

$$+ dq \times V''$$

Energy stored in a Capacitor

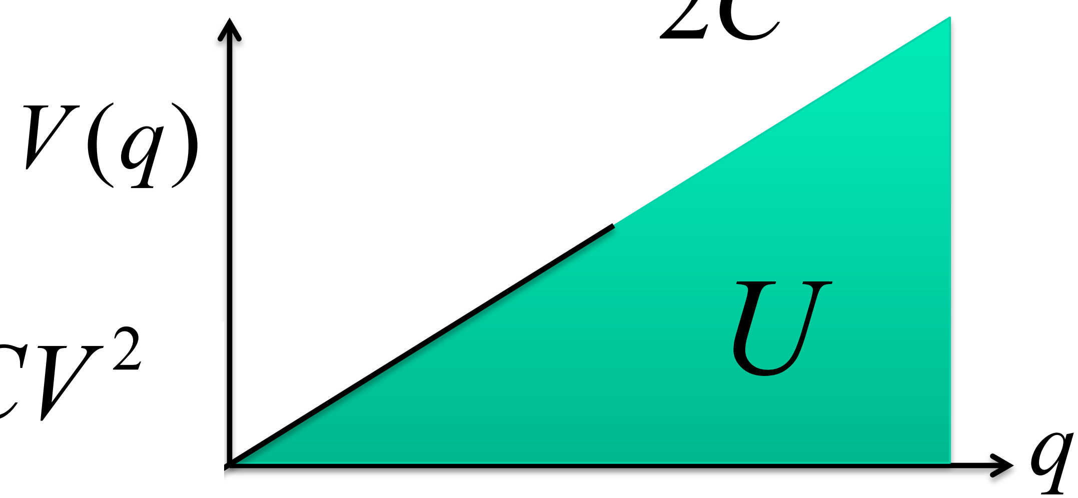


$$V(q) = \frac{q}{C}$$

$$U = \int dq \times V(q)$$

$$= \int dq \times \frac{q}{C}$$

$$= \frac{q^2}{2C}$$

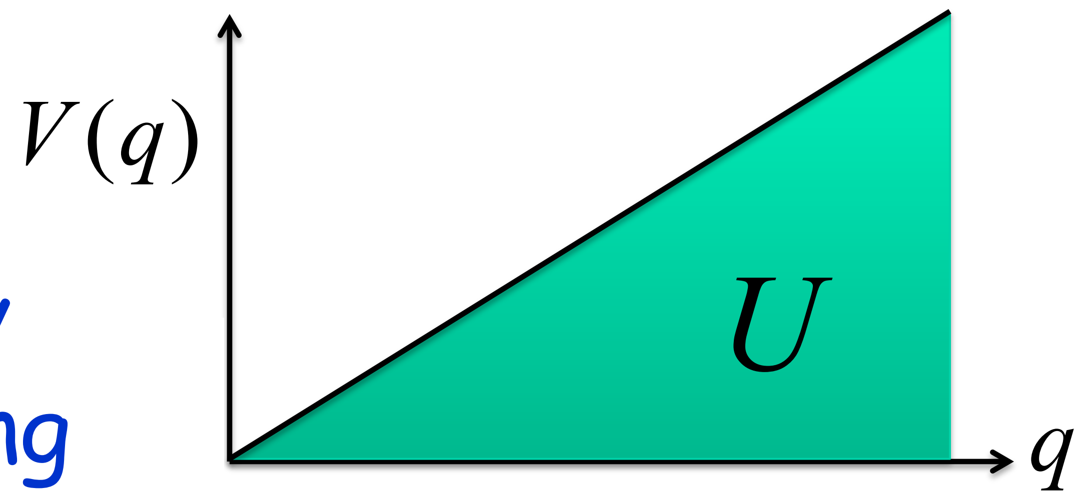


$$U = \frac{(CV)^2}{2C} = \tfrac{1}{2} CV^2$$

Energy stored in a Capacitor

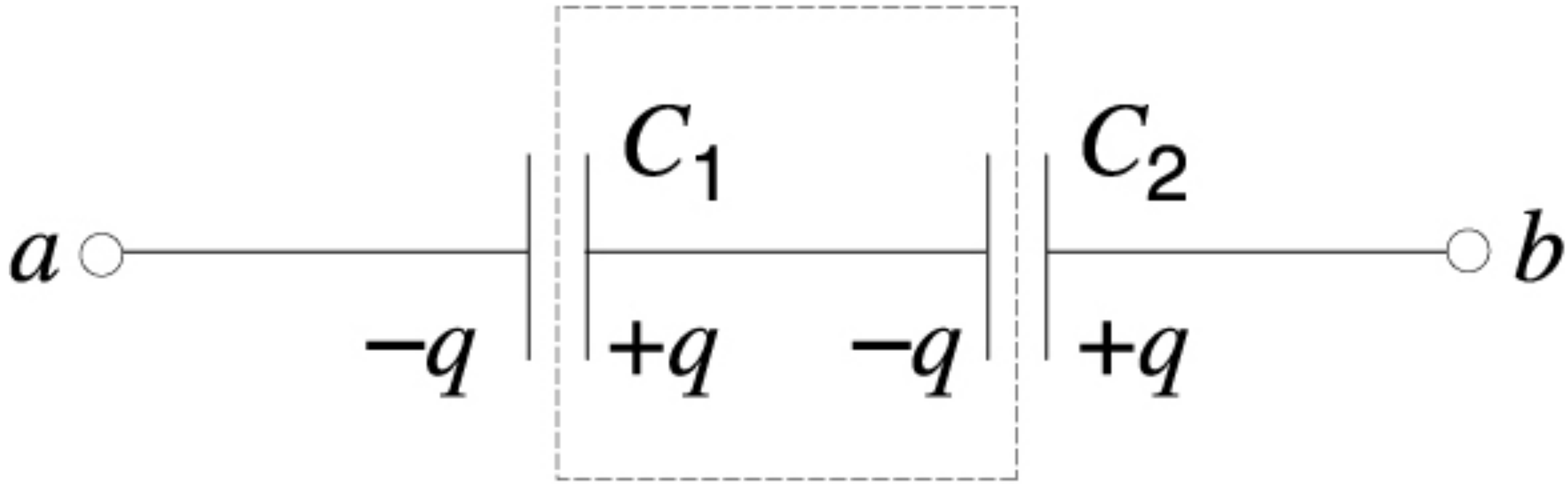
$$U = \frac{q^2}{2C} = \frac{(CV)^2}{2C} = \tfrac{1}{2}CV^2$$

$$= \frac{q^2}{2C} = \tfrac{1}{2}\frac{q}{C}q = \tfrac{1}{2}qV$$



Just like energy stored in a spring

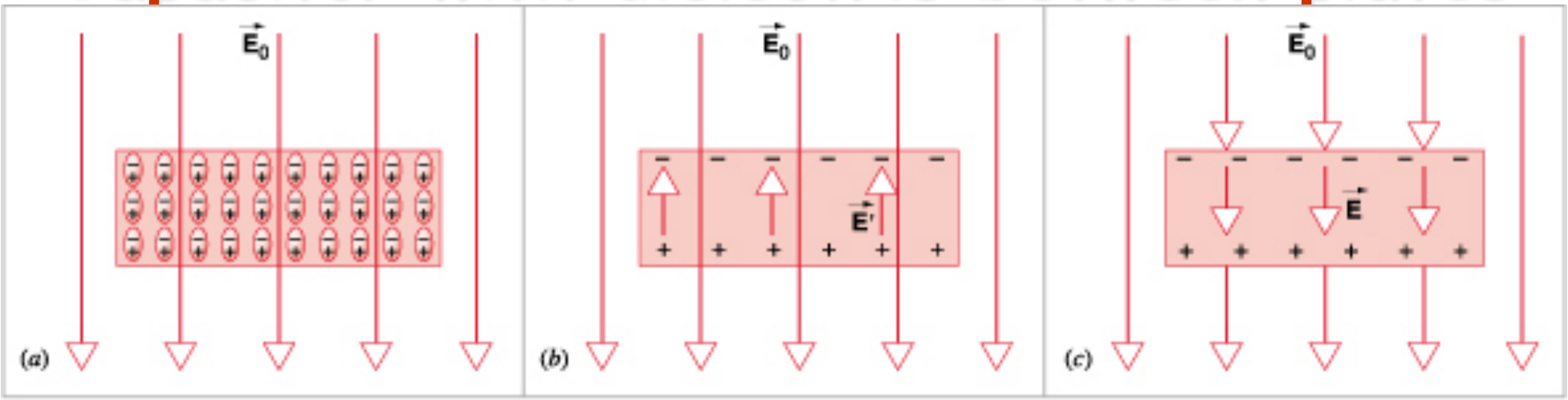
Capacitors connected in series



$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$$

In fact: $$\frac{1}{C_{eq}} = \sum_n \frac{1}{C_n}$$

Capacitor with dielectric between plates



Linear materials: $$E_0 = (1+\chi_e)E$$

Isolated capacitor: $$\Delta V = \frac{\Delta V_0}{\kappa_e} = \frac{1}{\kappa_e}\frac{Qd}{A\varepsilon_o}$$

Capacitance increases:

$$\Rightarrow \qquad C_{eff} = \frac{Q}{\Delta V} = \kappa_e \frac{\varepsilon_o A}{d} = \kappa_e C$$